



RELIABILITY

# Basic Probability Concepts

- Probability refers to the study of *randomness* and *uncertainty*.
- Probability theory provides methods for quantifying the chances or likelihoods associated with various outcomes.
- For example:
  - The probability of selling a certain number of products in a given year
  - The probability of a good part being produced
  - The reliability of a new component

Probability

Sample Space



1

2

3

4

5

6

Outcomes

Random  
Process

# Probability

- Random Process: any process whose possible results are known but whose actual results cannot be predicted with certainty in advance
- Outcome: each possible result for a random process
- Sample space: the set of all possible outcomes in an experiment
- Event: any collection (or subset) of outcomes contained in the sample space
  - Simple Event: an event that cannot be decomposed, one outcome of the experiment or in the sample space
  - Compound Event: collection of specified outcomes contained in the sample space
  - Null Event ( $\emptyset$ ): An event with no outcomes

# Types of Random Variables

- A *discrete* random variable is a random variable whose possible values either constitute a finite set or else can be listed in an infinite sequence.
- A random variable is *continuous* if its set of possible values consists of an entire interval on a number line (and is always infinite)

# Example

- Are the following random variables discrete or continuous?
  - The number facing up after a die roll      **Discrete**
  - The number of failed components an hour into an acceptance test      **Discrete**
  - The lifetime of a component      **Continuous**
  - The number of working components in a system with redundancy after 10 cycles      **Discrete**

# Probability Density Function (pdf)

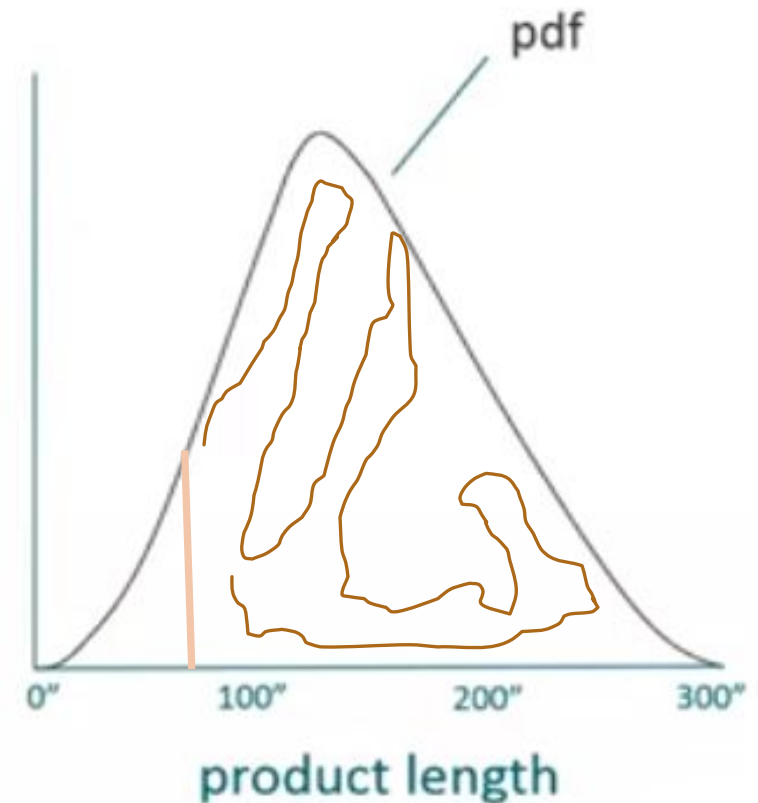
- Let  $X$  be a continuous rv. Then a probability density function (pdf) of  $X$  is a function  $f(x)$  such that for any two numbers  $a$  and  $b$ ,

$$P(a \leq X \leq b) = \int_a^b f(x)dx$$

- The graph of  $f(\cdot)$  is the density curve.

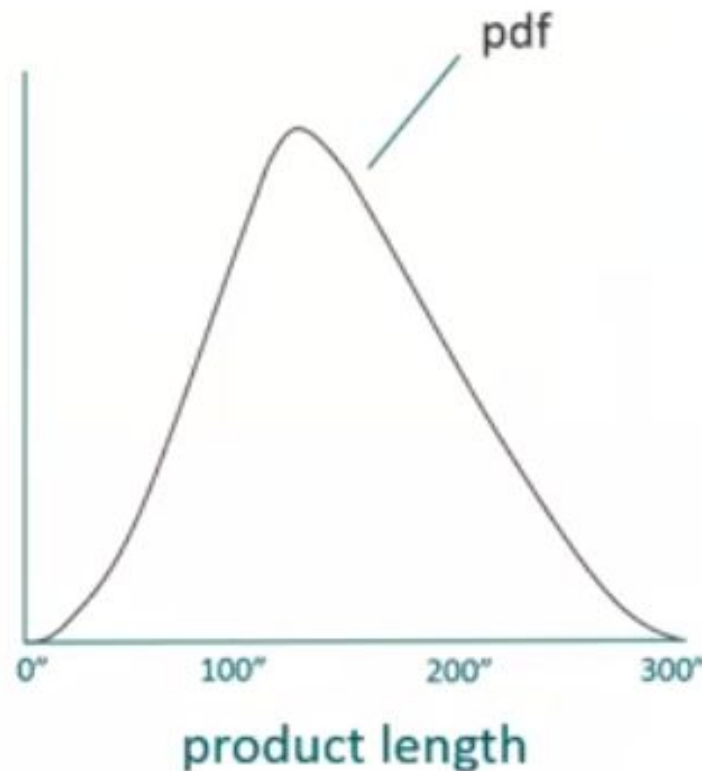
# Probability Density Function (pdf)

- What is the probability of a single product, pulled randomly from the population, having length
  - $> 95''$



# Probability Density Function (pdf)

- For  $f(x)$  to be a pdf
  - $f(x) \geq 0$  for all values of  $x$
  - the area of the region between the graph of  $f$  and the  $x$  – axis is equal to 1, i.e.,  $\int_{-\infty}^{\infty} f(x)dx = 1$



# Example

- Suppose that  $X$  is a continuous rv whose pdf is given by

$$f(x) = \begin{cases} C(4x - 2x^2), & 0 < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

What is the value of  $C$ ?

$$\int_{-\infty}^{+\infty} f(x) dx = \int_{-\infty}^0 f(x) dx + \int_0^2 f(x) dx + \int_2^{+\infty} f(x) dx = 1$$

$$\int_0^2 C(4x - 2x^2) dx = 1$$

$$\Rightarrow C = \frac{3}{8}$$

## Probability for a Continuous rv

- If  $X$  is a continuous rv, then for any number  $c$ ,

$$P(X = c) = 0.$$

That is, we cannot assign a positive probability to each of infinitely possible points

- Therefore, for any two numbers  $a$  and  $b$  with  $a < b$ ,

$$P(a \leq X \leq b) = P(a < X \leq b)$$

$$= P(a \leq X < b)$$

$$= P(a < X < b)$$

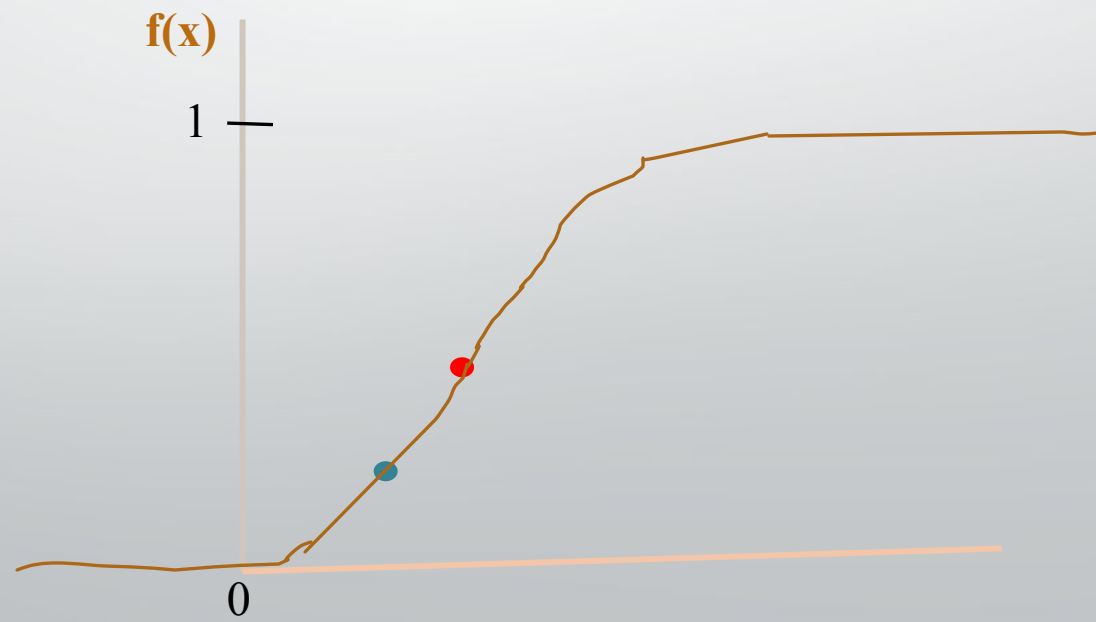
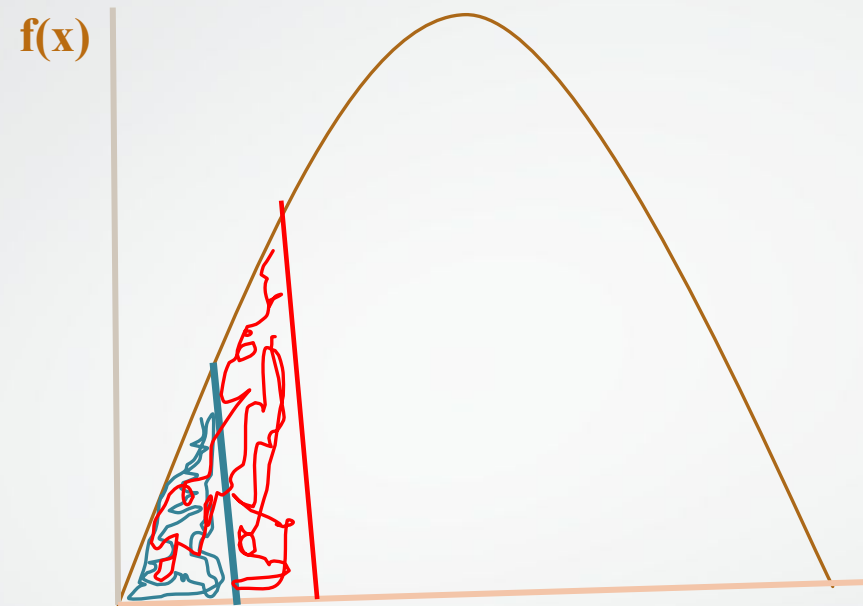
## Cumulative Distribution Function (CDF)

- The cumulative distribution function,  $F(x)$  for a continuous rv  $X$  is defined for every number  $x$  by

$$F(x) = \int_{-\infty}^x f(x)dx$$

- For each  $x$ ,  $F(x)$  is the area under the density curve to the Left of  $x$

# Cumulative Density Function



# Example

- Time to failure (in hours) for a capacitor has the following pdf. What is the probability of failure by  $t=200$  hr?
- $f(t)=0.01 e^{-0.01t}$

Since it's time hence must start from zero (instead of infinity)

$$P(t < 200) = \int_0^{200} f(t)dt = \int_0^{200} 0.01e^{-0.01t}dt$$

$$= 0.865$$

# Reliability Function

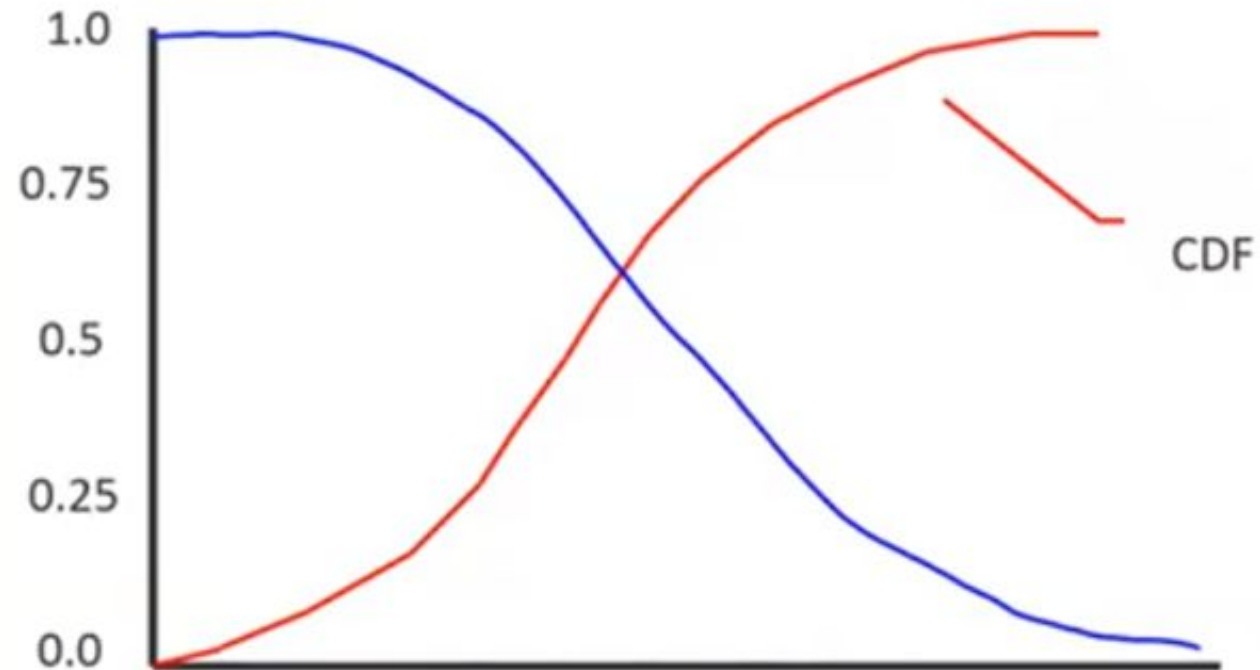
- Let random variable  $T$  be the time to failure of a system (or a component);  $T \geq 0$ . The reliability function is

$$R_T(x) = \int_x^{+\infty} f(x)dx = 1 - F(x)$$

- Boundaries of  $R(x)$ ?  **$R(0) = 1$     $R(+\infty) = 0$**
- For each  $x$ ,  $R(x)$  is the area under the density curve to the **Right** of  $x$

# Reliability Function

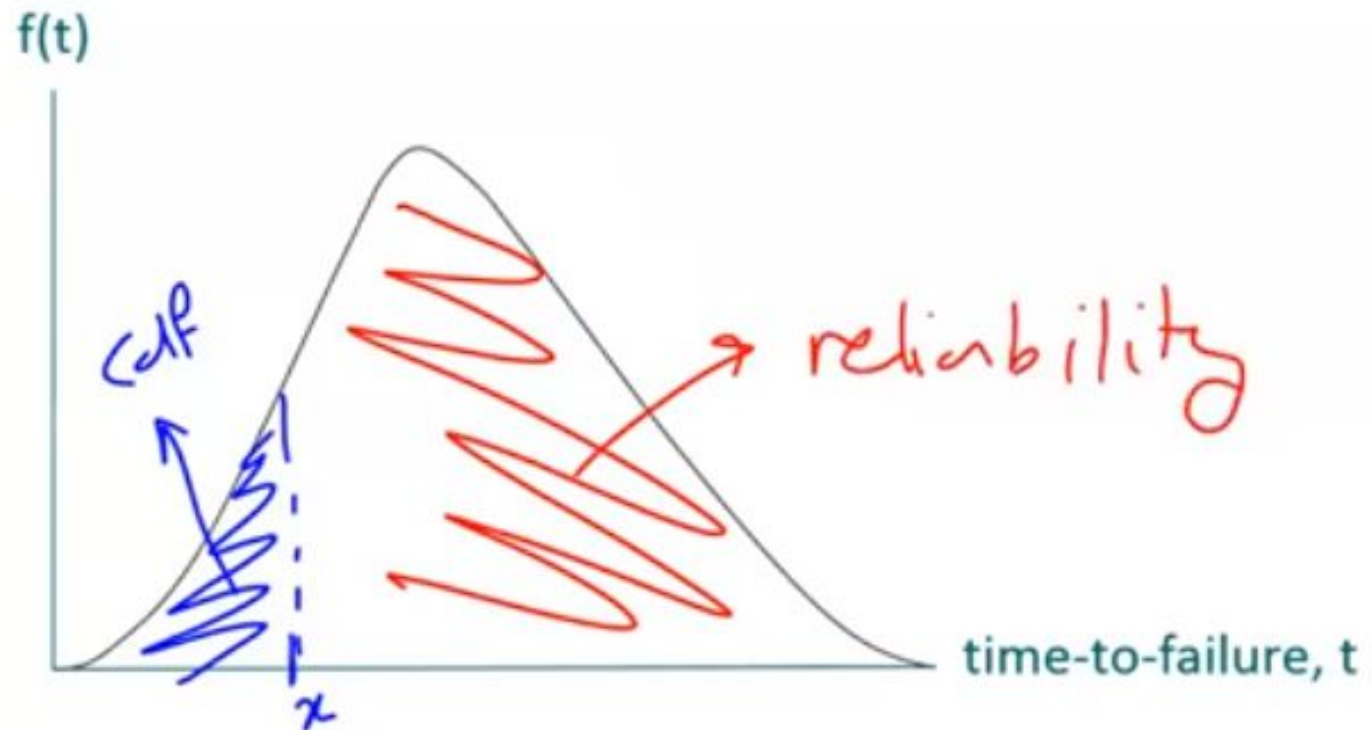
- Reliability function is the complement of CDF



$$R(x) + F(x) = 1$$

# Reliability Function

- If we are modeling time-to-failure:
  - What does the CDF represent?
  - What does the reliability function represent?



## Relationship Between $f(x)$ , $F(x)$ and $R(x)$

$$F(x) = \int_{-\infty}^x f(y) dy$$

$$R(x) = \int_x^{\infty} f(y) dy$$

$$f(x) = \frac{dF(x)}{dx}$$

$$F(x) = 1 - R(x) \quad \Rightarrow \quad \frac{dF(x)}{dx} = -\frac{dR(x)}{dx} = f(x)$$

